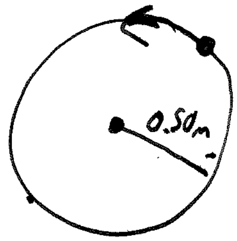
Uniform Circular Motion

Example: A mass connected to a rope is being swung around in a circle at a constant speed. Determine the speed of the mass if the rope is 0.50 m long, and it completes two rotations per second.



$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{0.5\text{sec}}$$

$$= \frac{2\pi 0.5\text{ m}}{0.5\text{sec}}$$

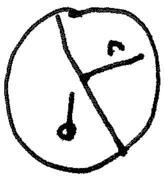
$$= 6.28\text{ m/s}$$

Is the acceleration of the mass zero?

No, velocity is changing

Terminology

Circumference: Length around a circle $= 2\pi r$ or πd



Period:

T = time to complete 1 revolution | $T = \frac{1}{f}$

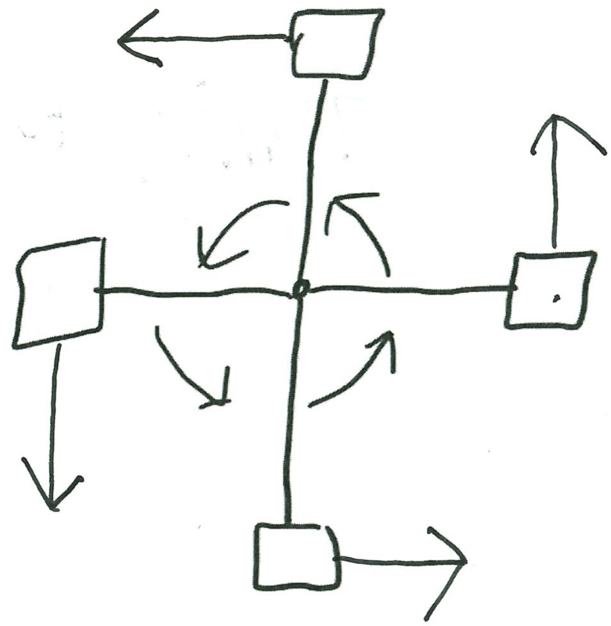
Frequency:

f = # of revolutions per second | $f = \frac{1}{T}$

Hz

Since acceleration is a change in velocity, and velocity is a vector, even though the speed is constant, since the direction of the movement is changing the acceleration is NOT zero.

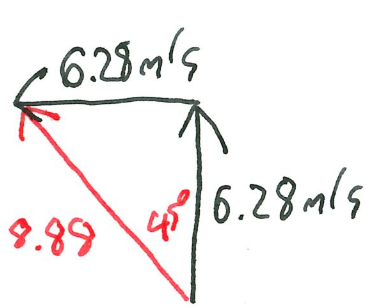
Consider the direction the mass is moving at various points as it goes around the circle:



Determine the acceleration of the mass as it moves from the southern most position to the easternmost position:

$$a = \frac{\Delta v}{t} = \frac{v_f - v_o}{t} = \frac{6.28\,m/s\ \text{North} - 6.28\,m/s\ \text{East}}{0.125\ \text{sec}}$$

$$= \frac{6.28\,m/s\ \text{North} + 6.28\,m/s\ \text{West}}{0.125\ \text{sec}}$$



$$= \frac{8.88\,m/s,\ 45^\circ\ \text{West of North}}{0.125\ \text{sec}}$$

$$= 71\,m/s^2,\ 45^\circ\ \text{West of North}$$

Determine the acceleration of the mass as it moves from the easternmost position to the northernmost position:

$71 m/s^2$, $45°$ West of South

Note that the acceleration is always directed toward the centre of the circle.

This is called centripetal acceleration. For an object moving at uniform circular motion:

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

speed

radius

radius

period

Using Newton's second law we can determine the centripetal FORCE that keeps the object moving.

$$F_{net} = ma \rightarrow F_c = ma_c = \frac{mv^2}{r} \text{ or } \frac{4\pi^2 rm}{T^2}$$

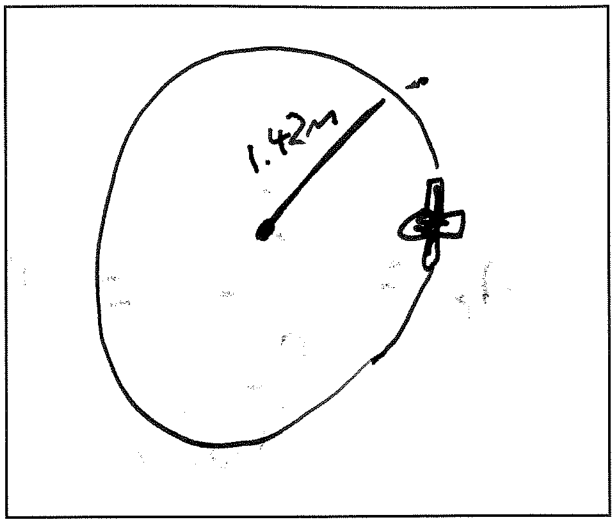
Centripetal force is not a new type of force, it is whatever force is being used to accelerate an object in circular motion

A swinging weight, the centipedal force is provided by tension in rope

In a car making a turn the centipedal force is provided by friction

When the moon orbits the earth the centipedal force is provided by gravity

EXAMPLE: A 2.5 kg toy plane is moving in uniform circular motion, completing a revolution around a circle of radius 1.42m every 6.3 seconds.



What is the period of the plane?

6.3sec

What is the frequency of the plane?

$$f = \frac{1}{T} = \frac{1}{6.3\,sec} = 0.16\,Hz$$

What is the speed of the plane?

$$speed = \frac{2\pi r}{T} = 2\pi r f = 1.4\,m/s$$

What is the velocity of the plane?

Dumb Question or 1.4m/s tangent to circle

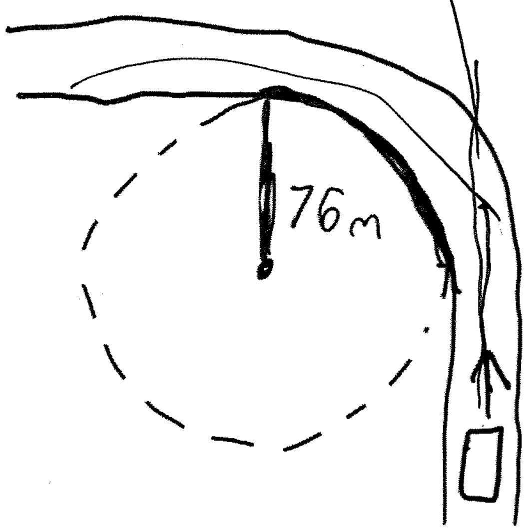
What is the acceleration of the plane?

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (1.42m)}{(6.3\,sec)^2} = 1.4\,m/s^2 \text{ towards centre}$$

What is the centripetal force acting on the plane?

$$F_c = m a_c = 2.5\,kg \times 1.4\,m/s^2 = 3.5\,N$$

EXAMPLE: A 1200 kg car goes around a curve of radius 76 m at 24 m/s. What is the magnitude of the centripetal force acting on the car?



$$F_c = \frac{mv^2}{r} = \frac{1200\,kg \times (24\,\frac{m}{s})^2}{76\,m}$$

$$= 9094.74\,N$$

What is the magnitude of the centripetal force if the car was moving at 35 m/s?

$$F_c = \frac{mv^2}{r} = \frac{1200 \times (35\,\frac{m}{s})^2}{76} = 19342.1\,N$$

What is the minimum coefficient of friction between the car's tires and the road under each scenario?

$$F_{fric} = F_c = 9094.74\,N$$

$$\mu = \frac{F_{fric}}{F_N} = \frac{9094.74}{1200 \times 9.8}$$

$$= 0.77$$

$$\mu = \frac{19342.1\,N}{1200 \times 9.8}$$

$$= 1.64$$

What will happen if the coefficient of friction is less than this?

You slide off road