Gravitational Potential Energy Practice Name:_______________________________

$m_{Earth} = 5.97 \times 10^{24}$kg $\quad r_{Earth} = 6371$ km

1. A 1500 kg satellite is launched into space from the surface of Earth.
   a. What is the gravitational potential energy (relative to infinite) of the satellite when it is on the surface of Earth?

$$E_p = -\frac{Gm_1m_2}{r} = -\frac{(6.674\times10^{-11})(1500kg)(5.97\times10^{24}kg)}{6371000m} \approx -9.381\times10^{10}J$$

$$= -9.4\times10^{10}J$$

   b. What is the gravitational potential energy (relative to infinite) of the satellite when it is 530 km above the surface of the Earth?

$$E_p = -\frac{(6.674\times10^{-11})(1500kg)(5.97\times10^{24}kg)}{(6371000m + 530000m)} = -8.66\times10^{10}J$$

$$= -8.7\times10^{10}J$$

   c. What is the minimum amount of work needed to raise the satellite to that height?

$$W = \Delta E = E_{pf} - E_{pi} = -8.66\times10^{10} - (-9.381\times10^{10})$$

$$= 7.2\times10^{9}J$$

   d. How much different is your answer from c to what you would get using $E_p = mgh$ with g = 9.8m/s^2?

$$E_p = 1500kg \times 9.8\frac{m}{s^2} \times 530000m$$

$$= \cancel{\quad}\times10^{9}$$

$$7.79\times10^{9}J - 7.2\times10^{9}J$$

Answer would be 590,000,000 J too much

Gravitational Potential Energy Practice Name:______________________________

$m_{Earth} = 5.97 \times 10^{24}$kg $r_{Earth} = 6371$ km

2. A 235 kg satellite is launched into space from the surface of Earth.
   a. What is the gravitational potential energy (relative to infinite) of the satellite when it is on the surface of Earth?

$$E_p = \frac{-Gm_1m_2}{r} = -1.47\times10^{10}\,J$$

   b. What is the gravitational potential energy (relative to infinite) of the satellite when it is 55 530 km above the surface of the Earth?

$$E_p = -\frac{Gm_1m_2}{r} = -1.51\times10^{9}\,J$$

   c. What is the minimum amount of work needed to raise the satellite to that height?

$$W = \Delta E = E_{pf} - E_{pi}$$
$$= -1.51\times10^{9}J - (-1.47\times10^{10}J)$$
$$= 1.32\times10^{10}\,J$$

   d. How much different is your answer from c to what you would get using $E_p = mgh$ with g = 9.8m/s^2?

$$E_p = mgh = 1.28\times10^{11}\,J$$

Almost 10 times too high using mgh

1.15×10^{11} too high

Gravitational Potential Energy Practice Name:______________________________

$m_{Earth} = 5.97 \times 10^{24}\text{kg}$ $r_{Earth} = 6371 \text{ km}$

3. A 2.0 kg hammer is placed 12 000 km above the surface of the Earth with an initial velocity of zero. Assuming no energy is lost to friction, how much kinetic energy will it have when it reaches the surface of Earth?

$$E_{p_i} = \frac{-6.674\times10^{-11} \times 2.0 \times 5.97\times10^{24}}{(18\,371\,000)} = -4.34\times10^{7}\text{ J}$$

$$E_{p_f} = -1.25\times10^{8}\text{ J}$$

$$\Delta E_p = E_{p_f} - E_{p_i} = -1.25\times10^{8} - (-4.34\times10^{7}) = -8.2\times10^{7}\text{ J}$$

E_p lost is E_K gained

$$E_K = 8.2\times10^{7}\text{ J}$$

4. A 2.0 kg hammer is placed a very long distance away from Earth (treat the distance as being infinitely far) with an initial velocity of zero. Assuming no energy is lost to friction, how much kinetic energy will it have when it reaches the surface of Earth?

$$E_{p_i} = 0 \quad , \quad E_{p_f} = -1.25\times10^{8}\text{ J}$$

$$E_K = 1.25\times10^{8}\text{ J}$$

Gravitational Potential Energy Practice Name:________________________

$m_{Earth} = 5.97 \times 10^{24}$kg $r_{Earth} = 6371$ km

5. A 560 kg satellite is launched into space.
   a. What is the potential energy (relative to infinite) of the satellite when it is on the surface of Earth?

$$E_p = -\frac{6.674\times10^{-11} \times 560 \times 5.97\times10^{24}}{6371000}$$

$$= -3.50\times10^{10}\,J$$

   b. What is the potential energy of the satellite when it is 12 000 km above the surface of the planet?

$$E_p = -1.21\times10^{10}\,J$$

   c. If the satellite is in uniform circular motion during its orbit, how fast is it moving?

$$a_c = g$$

$$\frac{v^2}{\cancel{r}} = \frac{GM}{r^{\cancel{2}}}$$

$$v = \sqrt{\frac{GM}{r}} = \cancel{7908\,m/s}\ 4657$$

   d. How much kinetic energy does the satellite have when it is in orbit.

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(560)(\cancel{7908})^2$$

$$= \cancel{1.75\times10^{10}\,J}\ 6.07\times10^9$$

   e. How much work must be done to lift the satellite into its orbit and give it enough velocity to stay in the orbit?

$$\Delta E_p = -1.21\times10^{10} - (-3.50\times10^{10}) = 2.29\times10^{10}\,J$$

$$\Delta E_k = \cancel{1.75\times10^{10}\,J}\ 6.07\times10^9$$

$$\text{Total} = 2.29\times10^{10} + \cancel{1.75\times10^{10}} = (\cancel{4.0\times10^{10}\,J})\ 2.9\times10^{10}$$

Gravitational Potential Energy Practice Name:________________________________

$m_{Earth} = 5.97 \times 10^{24}$kg $r_{Earth} = 6371$ km

6. Two 1.0 kg balls of radius 0.10 metres are placed very far apart (treat as infinitely far apart) in empty space, each with an initial velocity of zero. How fast will each be moving when they finally collide with each other? They will be 0.10 m apart when they collide

$$E_{pi} = 0, \quad E_{pf} = -\frac{6.674\times10^{-11}\times1\times1}{0.1} = -6.674\times10^{-10}$$

E_K when they collide is 6.674×10^{-10} J

$$V = \sqrt{\frac{2E_K}{m}} = 3.7\times10^{-5}\ m/s$$

7. A person stands on an asteroid of radius 25 km which has a mass of 5.6×10^{15} kg. If they throw a baseball of mass 0.142 kg at a speed of 3.0 m/s upwards how far above the asteroid does it travel before it is pulled back by gravity?

It will be pulled back if $E_K = 0$

E_K is converted into E_p so when $\Delta E_p = E_{K\,initial}$

$$E_{Ki} = \frac{1}{2}(0.142)(3)^2 = 0.639\,J$$

$$E_{pi} \text{ on surface} = \frac{-6.674\times10^{-11}\times0.142\times5.6\times10^{15}}{25\,000}$$

$$= -2.12\ J$$

$$\Delta E_p = E_{pf} - E_{pi}$$

$$\Delta E_p + E_{pi} = E_{pf}$$

$$0.639J + (-2.12) = -1.481\ J$$

$$E_{pf} = \frac{-Gm_1m_2}{r}$$

$$r = \frac{-Gm_1m_2}{E_{pf}} = \frac{(-6.674\times10^{-11})(0.142)(5.6\times10^{15})}{-1.481}$$

$$= 35835\ m - 25\,000\ m = 10,835\ m$$

Gravitational Potential Energy Practice Name:______________________________

$m_{Earth} = 5.97 \times 10^{24}\text{kg}$ $r_{Earth} = 6371 \text{ km}$

8. A person stands on an asteroid of radius 25 km which has a mass of 5.6×10^{15} kg. If they throw a baseball of mass 0.142 kg at a speed of 25.0 m/s upwards how far above the asteroid does it travel before it is pulled back by gravity (or will it ever be pulled back)?

$E_K = \frac{1}{2}(0.142)(25)^2 = 44.375 \text{ J}$

E_{pi} on surface $= -2.12$ J

$E_{Ki} = \Delta E_p = E_{pf} - E_{pi}$

$44.375 \text{ J} + (-2.12 \text{ J}) = E_{pf}$

$42 = E_{pf}$ ← E_{pf} is always negative only reaching zero at infinity so it won't come back

9. A person stands on an asteroid of radius 25 km which has a mass of 5.6×10^{15} kg. What is the minimum speed they could throw a 0.142 kg baseball so the baseball NEVER comes back?

E_K must be 2.12 J so E_{pf} is zero

$v = \sqrt{\frac{2 E_K}{m}} =$ 5.5 m/s

Gravitational Potential Energy Practice Name:_______________________________

$m_{Earth} = 5.97 \times 10^{24}\text{kg}$ $r_{Earth} = 6371 \text{ km}$

10. What is the escape velocity for an object moving away from the surface of the Moon. The mass of the Moon is 7.34×10^{22} kg and its radius is 1737 km.

Escape velocity is the speed it needs to go to end up ifinitely far from the Moon, at that point $E_p = 0 \rightarrow \Delta E_p = 0 - (E_{pi})$

$$E_K = -E_{pi}$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{G\cancel{m_1}m_2}{r}$$

$$v = \sqrt{\frac{2GM}{r}} = 2370\text{m/s}$$

11. What velocity must an object be moving at Earth's distance from the Sun (151 million km) so that it will not return to the solar system? (Mass of the Sun is about 2.0×10^{30} kg)

$$v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times 6.674 \times 10^{-11} \times 2 \times 10^{30}}{151000000000}}$$

$$= 42\,000\text{m/s}$$

Gravitational Potential Energy Practice Name:________________________________

$m_{Earth} = 5.97 \times 10^{24}$kg $r_{Earth} = 6371$ km

12. A 250 kg satellite is orbiting in a circular orbit, 17 000 km above the surface of the Earth.
    a. What is the velocity of the satellite?

$$\frac{v^2}{r} = \frac{GM}{r^2} \rightarrow v = \sqrt{\frac{GM}{r}} = 4.129 \times 10^3 \, m/s$$

    b. How much kinetic energy does the satellite have?

$$E_K = \frac{1}{2} mv^2 = 2.13 \times 10^9 \, J$$

    c. How much gravitational potential energy (relative to infinite) does the satellite have?

$$E_p = -\frac{Gm_1m_2}{r} = -4.26 \times 10^9 \, J$$

    d. What is the total energy (both kinetic and potential relative to infinite) of the satellite?

$$\text{Total energy} = 2.13 \times 10^9 + (-4.26 \times 10^9)$$
$$= -2.13 \times 10^9 \, J$$

Gravitational Potential Energy Practice Name:________________________________

$m_{Earth} = 5.97 \times 10^{24}\text{kg}$ $r_{Earth} = 6371\text{ km}$

13. The satellite in question 12 is raised from its orbit of 17 000 km to an orbit of 25 000 km above the surface of the Earth.
    a. What is the new velocity of the satellite?

$$V = \sqrt{\frac{GM}{r}} = 3564\,m/s$$

    b. Did the velocity increase or decrease when its orbital radius increased?

decrease

    c. What is the new kinetic energy of the satellite?

$$E_K = \frac{1}{2}(250\,kg)(3564)^2 = 1.59\times10^9\,J$$

    d. What is the new potential energy relative to infinite of the satellite?

$$E_p = \frac{-Gm_1m_2}{r} = -3.175\times10^9\,J$$

    e. What is the new total energy of the satellite?

$$1.59\times10^9 + (-3.175\times10^9)$$

$$-1.585\times10^9\,J$$

    f. How much work must be done to raise the orbit of the satellite?

$$\Delta E = E_f - E_i = -1.585\times10^9 - (-2.13\times10^9)$$

$$= 5.4\times10^8\,J$$

Gravitational Potential Energy Practice Name:______________________________

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14 ~~16.~~ A 5500 kg rocket is launched upwards from Earth's surface, it outputs a constant force of 56 000 N, the rocket travels 650 km straight up above the surface of the planet. Ignoring air resistance, what is its speed at this point?

$$\text{Work done} = Fd = 56\,000\text{ N} \times 650\,000\text{ m}$$
$$= 3.64 \times 10^{10}\text{ J}$$

$$\Delta E_p = E_{pf} - E_{pi}$$
$$= \frac{-6.674\times10^{-11} \times 5500 \times 5.97\times10^{24}}{6371000 + 650000} - \frac{-6.674\times10^{-11} \times 5500 \times 5.97\times10^{24}}{6371000}$$
$$= -3.12\times10^{11} - (-3.44\times10^{11})$$
$$= 3.20\times10^{10}\text{ J}$$

This much work to lift
remainder to speed up

$$E_K = 3.64\times10^{10} - 3.20\times10^{10} = 4.4\times10^{9}\text{ J}$$

$$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{2\times4.4\times10^{9}}{5500}} =$$

1300 m/s