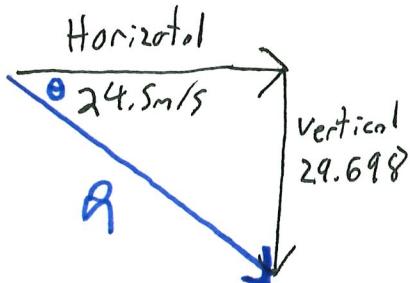


Projectile Motion Practice

Name: _____

Horizontal launch problems

1. A ball is launched horizontally at 24.5 m/s from a height of 45m.
 - a. What is the initial horizontal velocity of the ball? 24.5 m/s
 - b. What is the initial vertical velocity of the ball? 0 m/s
 - c. Does the horizontal velocity of the ball change while the ball is in the air? No, constant
 - d. Does the vertical velocity of the ball change while the ball is in the air? Yes
 - e. What is the vertical acceleration of the ball? -9.8 m/s^2
 - f. What will be the vertical displacement of the ball when it hits the ground? -45 m
 - g. Use the formula $d = v_0 t + \frac{1}{2} a t^2$ to determine how long it will take until the ball hits the ground.
- $\begin{matrix} -45 & 0 & -9.8 \end{matrix}$ $t = 3.030 \text{ sec}$
 $\approx 3.0 \text{ sec}$
- h. Use the formula $d = \bar{v}t$ to determine the horizontal displacement when the ball hits the ground.
- $\bar{v} = 24.5 \text{ m/s}$ $t = 3.030 \text{ sec}$ ← use not rounded value
- $d = 74.235 \approx 74 \text{ m}$
- i. Use the formula $v_f^2 = v_0^2 + 2ad$ to determine the final vertical velocity the ball hits the ground with.
- $\begin{matrix} 0 & -9.8 & -45 \end{matrix}$ negative because it is moving downwards
- $v_f = \pm 29.698 \dots$ $v_f = 3.0 \times 10^1 \text{ m/s}$
- j. Determine the final velocity (combined vertical and horizontal) the ball hits the ground with. Include both magnitude and direction.



$$\text{Magnitude} = \sqrt{24.5^2 + 29.698^2} \\ = 38.4996 \text{ m/s} \\ \approx 38 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{29.698}{24.5}\right) = 50.48^\circ$$

38 m/s, 5.0×10^1 degrees below the horizontal

Projectile Motion Practice

Name: _____

2. A ball is launched horizontally at 56m/s from a tower and hits a wall 250m horizontally away.

- a. Fill in the chart with the data given:

Horizontal	Vertical
$v_x = 56 \text{ m/s}$	$v_{oy} = 0 \text{ m/s}$
$d_x = 250 \text{ m}$	$a_y = -9.8 \text{ m/s}^2$

- b. Use the formula $d = v_x t$ to determine how long it will take until the ball hits the wall.

$$250 \text{ m} \quad 56 \text{ m/s}$$

$$t = 4.4643 \text{ sec}$$

$$\approx 4.5 \text{ sec}$$

- c. Use the formula $d = v_o t + \frac{1}{2} a t^2$ to determine the vertical displacement of the ball.

$$0 \quad -9.8 \text{ m/s}^2 \quad 4.4643 \text{ sec}$$

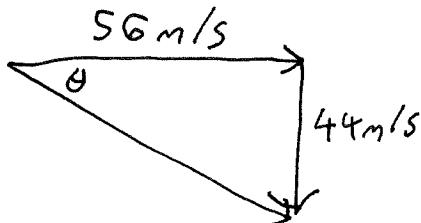
$$d = -97.6569 \text{ m} \approx -98 \text{ m}$$

- d. Use the formula $v_f = v_o + a t$ to determine the vertical velocity of the ball when it hits the wall.

$$0 \quad -9.8 \text{ m/s}^2 \quad 4.4643 \text{ sec}$$

$$v_f = -43.7501 \text{ m/s} \approx -44 \text{ m/s}$$

- e. Determine the velocity (combined horizontal and vertical) of the ball when it hits the wall. Include both magnitude and direction.



$$\text{Magnitude} = \sqrt{56^2 + 43.7501^2}$$

$$= 71 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{43.7501}{56}\right) = 38^\circ$$

71 m/s, 38° below the horizontal

Projectile Motion Practice

Name: _____

3. A gun is pointed horizontally, and a bullet is fired from it at 325 m/s. It is aimed at the centre of a target 100.0 m away. How far below the centre of the target will the bullet hit?

① Determine t : Use $d = \bar{v}t$

Using horizontal info

$$d = \bar{v}t$$

$$100.0 \text{ m} = 325 \text{ m/s} \cdot t$$

$$t = 0.30769 \text{ sec}$$

horizontal	vertical
$v_x = 325 \text{ m/s}$	$v_{oy} = 0 \text{ m/s}$
$d_x = 100.0 \text{ m}$	$a = -9.8 \text{ m/s}^2$
$t = 0.30769 \text{ s}$	looking to find d_y

② Determine vertical displacement

$$d_y = v_{oy}t + \frac{1}{2}at^2$$

$$0 \text{ m/s} \quad -9.8 \text{ m/s}^2 \quad 0.30769 \text{ sec}$$

$$d_y = -0.46 \text{ m}$$

0.46 m below centre

4. A ball is launched horizontally at a velocity of 19.4 m/s from a height of 4.3 m. How far horizontally from where it is launched will the ball land?

① Determine t using horizontal info : Use $d = v_0 t + \frac{1}{2} a t^2$

$$d = v_0 t + \frac{1}{2} a t^2$$

$$-4.3 \text{ m} = 0 + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2$$

X	Y
$v_x = 19.4 \text{ m/s}$	$v_{oy} = 0 \text{ m/s}$
$d_x = ?$	$a = -9.8 \text{ m/s}^2$
	$d_y = -4.3 \text{ m}$
	$t = 0.93678 \text{ s}$

$$t = 0.93678 \text{ sec}$$

② Determine horizontal displacement using $d = \bar{v}t$

$$d = 19.4 \text{ m/s} \cdot 0.93678 \text{ sec}$$

$d = 18 \text{ m}$

Projectile Motion Practice

Name: _____

5. A ball is launched horizontally from a height of 5.9 m, and hits the ground 2.35 metres horizontally from where it was launched. What was the initial horizontal velocity of the ball?

① Use vertical info to find t : Use $d = v_0 t + \frac{1}{2} a t^2$

$$d = v_0 t + \frac{1}{2} a t^2$$

$$-5.9 = 0t + \frac{1}{2} (-9.8)t^2$$

$$-5.9 = -4.9t^2$$

$$t^2 = \frac{5.9}{4.9}$$

$$t = \sqrt{\frac{5.9}{4.9}} = 1.0973 \text{ sec}$$

X	Y
$d_x = 2.35 \text{ m}$	$v_{0y} = 0 \text{ m/s}$
$v_x = ?$	$a_y = -9.8 \text{ m/s}^2$
	$d_y = -5.9 \text{ m}$
	$t = 1.0973 \text{ sec}$

② Determine horizontal velocity: Use $d = \bar{v} t$

$$d = \bar{v} t$$

$$2.35 = \bar{v} \cdot 1.0973$$

$$\bar{v} = \frac{2.35}{1.0973} = 2.1 \text{ m/s}$$

6. A ball is launched horizontally from a tower and hits the ground 25 metres horizontally from where it was launched, 4.3 seconds after it was launched. What is the velocity the ball hit the ground with (magnitude and direction)?

① Determine horizontal velocity

$$d = \bar{v} t$$

$$25 = \bar{v} \cdot 4.3$$

$$\bar{v} = \frac{25}{4.3} = 5.81395 \text{ m/s}$$

X	Y
$d_x = 25 \text{ m}$	$v_{0y} = 0 \text{ m/s}$
$v_x = 5.81395 \text{ m/s}$	$a_y = -9.8 \text{ m/s}^2$
	$t = 4.3 \text{ sec}$
	$v_f = -42.14 \text{ m/s}$

② Determine final vertical velocity

$$v_f = v_0 + a t$$

$$0 + (-9.8) \cdot 4.3 = -42.14 \text{ m/s}$$

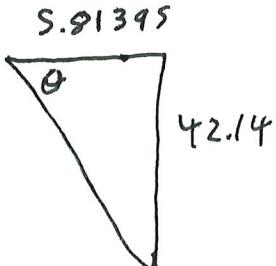
$$v_f = -42.14 \text{ m/s}$$

③ Determine resultant

$$\text{Magnitude} = \sqrt{(5.81395)^2 + (42.14)^2}$$

$$= 43 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{42.14}{5.81395}\right) = 82^\circ$$



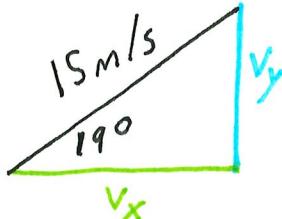
43 m/s, 82° below the horizontal

Projectile Motion Practice

Name: _____

Angled launch at 15m/s

7. A ball is launched from a height of 23m, at an angle of 19° above the horizontal.
- Determine the vertical and horizontal components of the initial velocity.



$$\sin 19 \times 15 = v_y$$

$$4.8835 \text{ m/s}$$

$$\cos 19 \times 15 = v_x$$

$$14.1828 \text{ m/s}$$

- What is the vertical acceleration?

$$-9.8 \text{ m/s}^2$$

- What will be the vertical displacement when the ball hits the ground? -23 m

- Use the formula $v_f^2 = v_0^2 + 2ad$ to determine the final velocity of the ball.

$$4.8835 \text{ m/s} \quad -9.8 \text{ m/s}^2 \quad -23 \text{ m}$$

$$v_{fy} = -21.7864 \text{ m/s}$$

$$\approx -22 \text{ m/s}$$

Going down so must be negative

- Use the formula $v_f = v_0 + at$ to determine how long the ball was in the air.

$$-21.7864 \text{ m/s} \quad -9.8 \text{ m/s}^2 \quad + = 2.7214 \text{ sec}$$

$$\approx 2.7 \text{ sec}$$

- Use the formula $d = \bar{v}t$ to determine how far horizontally the ball travels in that time.

$$14.1828 \text{ m/s} \quad 2.7214 \text{ sec}$$

$$d_x = 38.5971$$

$$\approx 39 \text{ m}$$

- What will the vertical velocity of the ball be when it is at its highest point?

$$0 \text{ m/s}$$

- Use the formula $v_f^2 = v_0^2 + 2ad$ to determine the displacement of the ball at its highest point.

$$0 \quad 4.8835 \text{ m/s} \quad -9.8 \text{ m/s}^2$$

$$d = 1.2 \text{ m}$$

- How high above the ground will the ball be at its highest point?

$$23 \text{ m} + 1.2 \text{ m} = 24 \text{ m}$$

Projectile Motion Practice

Name: _____

8. A ball is kicked from the ground at an angle 34 degrees above the horizontal with speed of 12 m/s. How far down the field will the ball land?

$$\begin{array}{l} \text{Diagram: A right triangle with hypotenuse } 12 \text{ m/s, angle } 34^\circ \text{ from the horizontal.} \\ \text{Vertical component: } \sin 34^\circ \times 12 = 6.710 \text{ m/s} \\ \text{Horizontal component: } \cos 34^\circ \times 12 = 9.948 \text{ m/s} \end{array}$$

① Find t using: $v_f = v_0 + at$

vertical info: $v_f = -6.71 \text{ m/s}$, $v_0 = 6.71 \text{ m/s}$, $a = -9.8 \text{ m/s}^2$

$$-6.71 = 6.71 + (-9.8)t$$

$$-13.42 = -9.8t$$

$$t = 1.369 \text{ sec}$$

X	Y
$v_x = 9.948 \text{ m/s}$	$v_{oy} = 6.710 \text{ m/s}$
	$a_y = -9.8 \text{ m/s}^2$
	$v_{fy} = -6.710 \text{ m/s}$
	↑ Since it falls to same height it started with $t = 1.369 \text{ sec}$

② Use $d = \bar{v}t$ to find horizontal displacement

$d = 9.948 \text{ m/s} \times 1.369 \text{ sec}$

$d = 14 \text{ m}$

9. A cannon at ground level shoots a cannonball at an angle of 76 degrees above the horizontal with speed of 56 m/s. What is the maximum height the cannonball will reach?

$$\begin{array}{l} \text{Diagram: A right triangle with hypotenuse } 56 \text{ m/s, angle } 76^\circ \text{ from the horizontal.} \\ \text{Vertical component: } \sin 76^\circ \times 56 = 54.34 \text{ m/s} \\ \text{Horizontal component: } \cos 76^\circ \times 56 = 13.55 \text{ m/s} \end{array}$$

X	Y
$v_x = 13.55 \text{ m/s}$	$v_{oy} = 54.34 \text{ m/s}$
	$a_y = -9.8 \text{ m/s}^2$
	$v_{fy} = 0$
	↑ at highest point

Find d_y using $v_f^2 = v_0^2 + 2ad$

$0 = 54.34^2 + 2(-9.8)d$

$2541.156 = -19.6d$

$d = 130.65 \approx 130 \text{ m}$

$$d = 150.65 \approx 150 \text{ m}$$

Projectile Motion Practice

Name: _____

10. A rock is thrown at an angle of 25 degrees above the horizontal at 13m/s from a 22 metre tall cliff. What will the final velocity of the rock (magnitude and direction) be?

$$\begin{aligned} \sin 25 \times 13 &= 5.494 \text{ m/s} \\ \cos 25 \times 13 &= 11.782 \text{ m/s} \end{aligned}$$

X	Y
$v_x = 11.782 \text{ m/s}$	$v_{oy} = 5.494 \text{ m/s}$
	$a = -9.8 \text{ m/s}^2$
	$d = -22 \text{ m}$

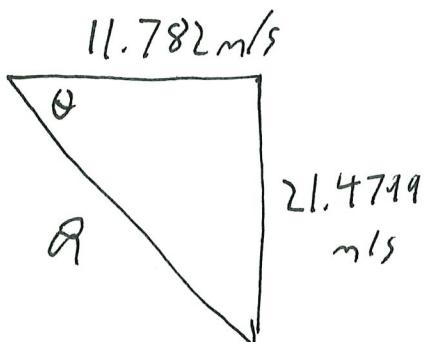
① Find vertical final velocity

$$v_f^2 = v_0^2 + 2ad$$

\uparrow \nwarrow
 5.494 m/s -9.8 m/s^2

$$v_f = -21.4799 \text{ m/s}$$

② Find resultant velocity



$$\text{Magnitude} = \sqrt{11.782^2 + 21.4799^2}$$

$$= 24.499 \text{ m/s}$$

$$\approx 24 \text{ m/s}$$

$$\theta = \tan^{-1} \left(\frac{21.4799}{11.782} \right)$$

$$= 61^\circ$$

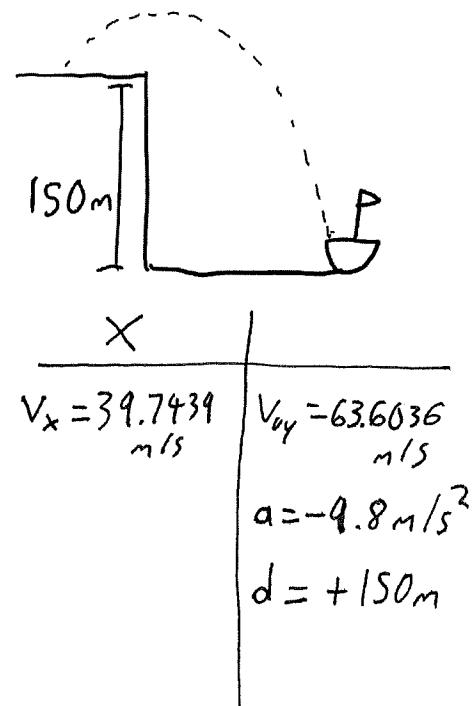
24 m/s, 61° below the horizontal

Projectile Motion Practice

Name: _____

11. A ship fires its cannon so that the cannonball lands at the top of a 150.0 m cliff. How far horizontally from the ship will the cannonball land if it is fired at 75 m/s at an angle of 58 degrees above the horizontal.

$$\begin{aligned} \sin 58 \times 75 &= 63.6036 \text{ m/s} \\ \cos 58 \times 75 &= 39.7439 \text{ m/s} \end{aligned}$$



Two methods could be used

① Use quadratic formula
with $d = v_0 t + \frac{1}{2} a t^2$
to find t

$$0 = \frac{1}{2} a t^2 + v_0 t - d$$

$$"a" = -4.9 \quad "b" = 63.6036$$

$$"c" = -150$$

use quadratic

$$t = 3.0975 \text{ or } 9.8828$$

Going up

Going down

② Use $d = \bar{v} t$ to find horizontal displacement

$$d = 39.7439 \times 9.8828 \text{ s}$$

$$= 392.78 \text{ m}$$

$$\approx 390 \text{ m}$$

Avoid quadratic formula

① Determine v_f using $v_f^2 = v_0^2 + 2ad$

$$v_f = \pm 33.2478 \text{ m/s}$$

② Determine times using
 $v_f = v_0 + at$

$$+ = 3.0975 \text{ if } v_f = +33.2478$$

$$+ = 9.8828 \text{ if } v_f = -33.2478$$

③ We use time when cannonball is falling

$$d = \bar{v} t$$

$\nearrow 39.7439 \quad \nwarrow 9.8828$

$$d = 390 \text{ m}$$